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Investigation of Trade-Off Between Bandwidth and Sidelobe Level for Convex Optimization of Arrays

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Summary

Using a convex optimization program for wideband arrays, a trade-off between optimisation bandwidth and sidelobe-level (SLL) is shown. The excitation coefficients are constrained to ensure that the solution can be realized in a wideband active electronically scanned array (AESA). Contrary to single-frequency optimization, the wideband optimization method used here ensures that the computed excitation is optimal over a specified bandwidth. The optimization program is implemented in terms of embedded element patterns to compensate for mutual coupling and edge effects. For a range of bandwidths, the loss of SLL suppression is investigated for an optimisation of a smaller than desired bandwidth. This shows that a worsening of SLL is to be expected for a single or small optimisation bandwidth, as compared to a full bandwidth optimisation.

1 Introduction

Several methods exist for determining optimal excitation weights for array antennas. Most of these methods are designed for optimising at a single frequency, e.g. [1]–[3]. This is usually deemed sufficient for applications such as communication systems. However, for electronic warfare and certain radar applications, there is a demand for larger bandwidths. Further, the results of this investigation show that even relatively narrow bandwidths can benefit from optimising over a frequency range. One way to apply single frequency optimisation to a wideband problem is to simply optimise for e.g. the center frequency, though this is expected to cause degraded performance toward the band edges. Another way is to apply the single-frequency optimisation at several frequency samples in the band, but this has been shown to result in a rapidly fluctuating phase across the frequency band [4]. Such a phase is difficult to implement in practical systems. This presentation will instead utilise a wideband optimisation method that has realistic constraints on both the amplitude and phase of the excitation coefficients, built on the method in [1]. Contrary to single-frequency optimization, the wideband optimisation method ensures that the computed excitation is optimal over a specified bandwidth. Results for optimised SLL for wideband arrays will be shown, together with studies of the trade-off between bandwidth and SLL.

2 Theory

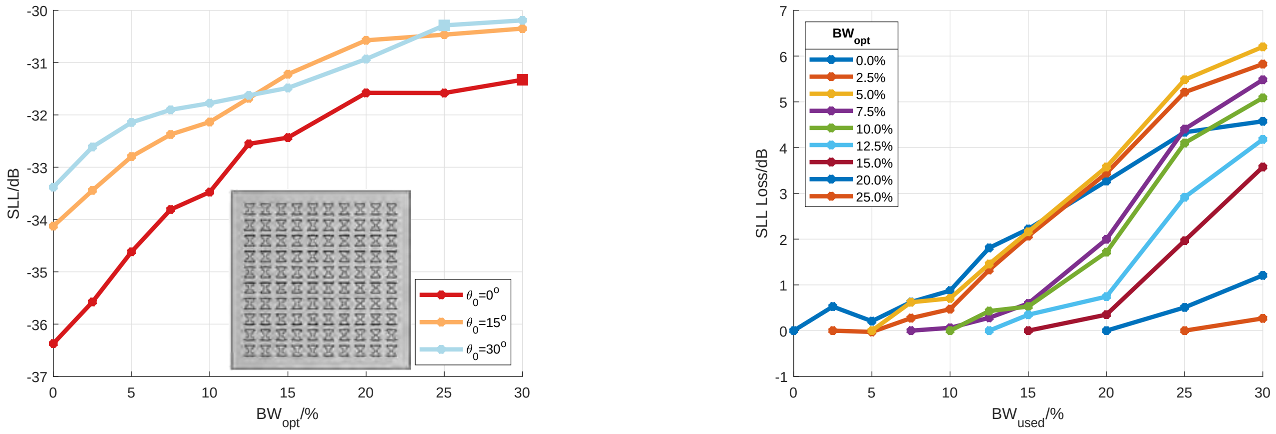
The optimisation method is formulated on embedded element patterns (EEPs) to account for mutual coupling and edge effects [3], [4]. The far-field $\vec{\mathbf{F}}$ of the array is given by the sum of all N EEPs $\vec{\mathbf{f}}_n$, with excitation coefficients a_n found by optimisation:

$$\vec{\mathbf{F}}(\hat{r},\omega) = \sum_{n=1}^{N} a_n(\omega)\vec{\mathbf{f}}_n(\hat{r},\omega), \tag{1}$$

where $\hat{r}$ is the direction in the radiation pattern, n the element index and ω the angular frequency. The excitation coefficient is given by an amplitude and a phasor: $a_n(\omega) = \alpha_n(\omega)e^{j\Phi_n(\omega)}$. To get a practically implementable excitation coefficient, it is assumed that a_n is varied using a variable gain amplifier with a gain slope (linear in dB) and Φ_n is varied using a phase shifter and a true time delay. For the SLL optimisation, $\vec{\mathbf{F}}$ is sampled at each degree in θ and ϕ at each frequency sample over the chosen optimisation bandwidth BW_{opt}. The main lobe is only considered for the co-polar component of $\vec{\mathbf{F}}$, while sidelobes are included for both the co- and cross-polar component. A convex optimisation problem is formed for the excitation coefficients. Optimal coefficients are found using a cost function of the SLL and CVX [5]. Details will be described in a forthcoming paper.

3 Results

The here investigated array is a bowtie array over a ground plane with 10×10 elements shown insterted in Figure 1a. Figure 1a shows the optimal SLL within a range of bandwidths. For both plots in Figure 1, N starts at 1 for the single frequency case and grows by 2 for each widening in bandwidth. SLL is only considered within BW_{opt} in Figure 1a. This result concerns the



(a) Optimal SLL for the array for varying optimisation bandwidths and for three different steering directions. θ_0 is degrees from boresight in the H-plane. Points marked with a square did not reach th CVX convergence criteria. Insert: array geometry.

(b) SLL loss over desired bandwidth for different optimisation bandwidths and $\theta_0 = 15^\circ$. For a chosen optimisation bandwidth, the sidelobe levels will degrade within the desirable bandwidth by the level shown.

Figure 1. Optimisation results.

trade-off between SLL and BW_{opt}, and the stability of the result with regards to scan angle (θ_0) in the H-plane. All bandwidths are defined around a common center frequency. While there are some variations, the optimal SLL is higher (i.e. worse) for higher bandwidths. This is expected as the optimisation problem grows, while the degrees of freedom stays constant. CVX did not meet its convergence criteria for two data points, marked with a square. These values may not be optimal. In general, a higher SLL is seen for a larger θ_0, except for between 15 % to 20 %, where $\theta_0 = 30^\circ$ has a lower SLL than $\theta_0 = 15^\circ$.

In Figure 1b, SLL is investigated for a bandwidth BW_{used} which is larger than BW_{opt} for $\theta_0 = 15^\circ$ in the H-plane. This is shown as an SLL loss, defined as the increase in SLL as $BW_{used} > BW_{opt}$. I.e. the difference between SLL for BW_{used} and for BW_{opt}. For example, consider the 0% optimisation bandwidth, i.e. single frequency. If the desired bandwidth is 5%, SLL is slightly higher (less than 1 dB) compared to an optimisation bandwidth of 5%. At a desired bandwidth of 30%, the single frequency optimisation loses almost 5 dB as compared to a full bandwidth optimisation. In comparison, the increase in SLL for increasing BW_{opt} from 0% to 30% in Figure 1a for $\theta_0 = 15^\circ$ is smaller than 4 dB. Optimisation time is observed to grow close to linearly with the number of frequency samples in the bandwidth. The SLL loss Figure 1b further depends on θ_0. For boresight, there is almost no dependency on the investigated desired bandwidth with a maximum loss of 1.5 dB. Increasing to $\theta_0 = 30^\circ$, the loss curve is sharper, with the single frequency optimisation losing almost 3 dB at 7.5 % desired bandwidth.

4 Acknowledgements

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